Province of the
EASTERN CAPE
EDUCATION

| LEARNER'S NAME: *LEERDER SE NAAM:* | SLT Solns |
|---|---|

| GRADE 11 *GRAAD 11* | |
|---|---|

# NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

## GRADE 11/*GRAAD 11*

## NOVEMBER 2023

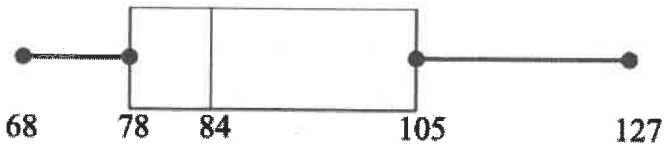
## MATHEMATICS P2/*WISKUNDE V2*
## SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

| Marker/*Merker* | | | | Moderator's Initials / *Moderator se paraaf* | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Question *Vraag* | Mark *Punt* | | | Initial Parafeer | Marks *Punte* | | | S M | Marks *Punte* | | | D M | Marks *Punte* | | | P M | Marks *Punte* | | | NM |
| 1 | | | | | | | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | | | | | | | |
| TOTAL *TOTAAL* | | | | | | | | | | | | | | | | | | | | |

This special answer book consists of 20 pages.
*Hierdie spesiale antwoordeboek bestaan uit 20 bladsye.*

* I M A T 4 *

**QUESTION 1/*VRAAG 1***




24 boys
$M = 84$
$\bar{x} = 87$

| | | |
|---|---|---|
| 1.1 | $\frac{25}{100} \times 24 = 6$ ✓ | 1 (1) |
| 1.2 | $\bar{x} - M = 87 - 84 = +3$ OR $105 - 84 = 21$ $84 - 78 = 6$ $21 > 6$<br>Skewed to the right (positively) ✓ | 1 (1) |
| 1.3 | Range $= 127 - 68 = 59$ marks ✓ ✓ | 2 (2) |
| 1.4.1 | $M = 84$ ✓ : remains the same | 1 (1) |
| 1.4.2 | $\bar{x}_{new} = 87,83$ ✓✓ : it has increased | 2 (2) |
| | | [7] |

$127 \rightarrow 147$

$\bar{x}$ : $24 \times 87 = 2088$

$- 127 + 147$

$= 2108$

$\bar{x}_{new} = \frac{2108}{24}$

$= 87,83$

OR

$\bar{x}$ : $87 + \frac{147 - 127}{24}$

$= 87 + \frac{20}{24}$

$= 87,83$

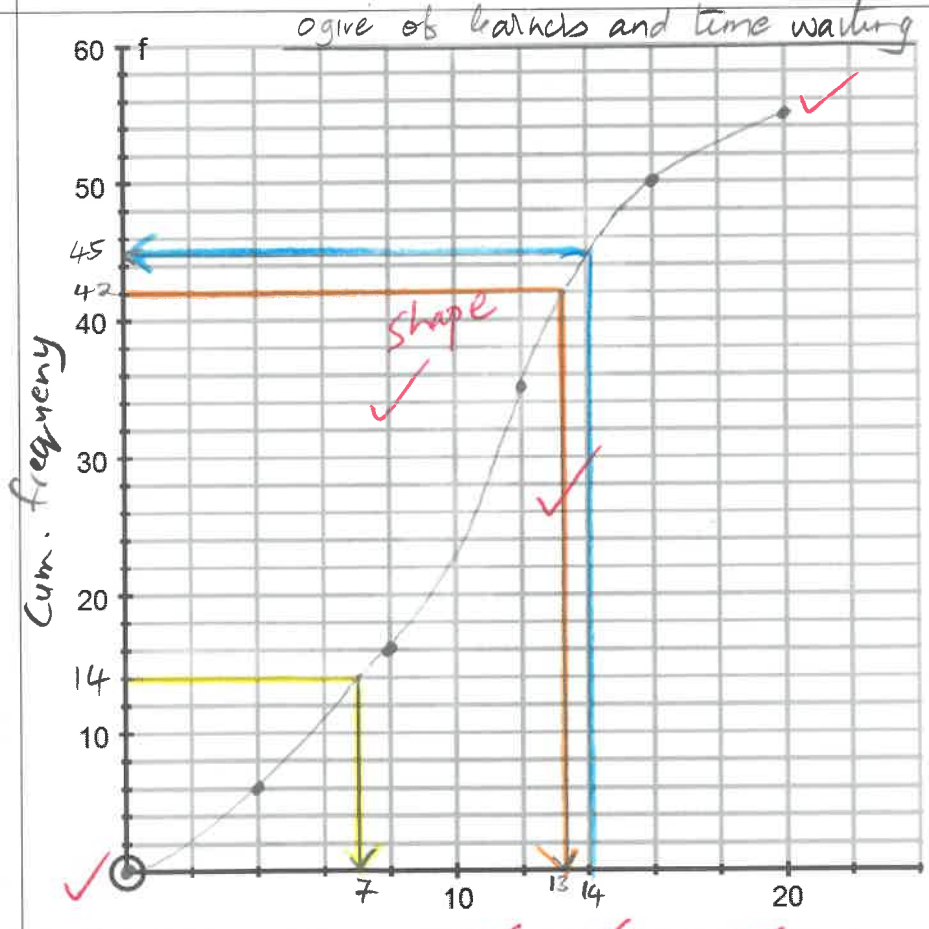
## QUESTION 2/*VRAAG 2*

**2.1**

| Time waiting / *Wagtyd* (minutes / *minute*) | Number of learners *Aantal leerders* | Cumulative frequency *Kumulatiewe frekwensie* |
|---|---|---|
| $0 < x \leq 4$ | 6 | 6 |
| $4 < x \leq 8$ | 10 | 16 ✓ |
| $8 < x \leq 12$ | 19 | 35 |
| $12 < x \leq 16$ | 15 | 50 ✓ |
| $16 < x \leq 20$ | 5 | 55 |

2 (2)

**2.2**




4 (4)

**2.3**

$>14 = 55 - \leq 14 = 55 - 45 = 10$ learners ✓

3 (3)

**2.4**

Modal class : $8 < x \leq 12$ ✓

1 (1)

**2.5**

| | | | |
|---|---|---|---|
| IQR = 13 − 7 ✓ | $T_1; \ldots; T_{55}$ | $T_1; \ldots; T_{27}$ | $T_{29}; \ldots; T_{55}$ |
| $\therefore$ 6 ✓ | $M = T_{28}$ | $Q_1 = T_{14} = 7$ | $Q_3 = T_{42} = 13$ |

3 (3)

**[13]**

$\frac{1}{2}(1+55) = 28$ $\frac{1}{2}(1+27) = 14$ $\frac{1}{2}(29+55) = 42$

**QUESTION 3/*VRAAG 3*** $A(-8;0)\quad B(x-5;-8)\quad C(x;-14)$

**3.1**

$M_{AB} = \frac{-8-0}{x-5-(-8)} = \frac{-8}{x-5+8} = \frac{-8}{x+3}$ ✓

$A_{BC} = \frac{-14-(-8)}{x-(x-5)} = \frac{-14+8}{x-x+5} = \frac{-6}{5}$ ✓

Collinear $\therefore \frac{-8}{x+3} = \frac{-6}{5}$ ✓

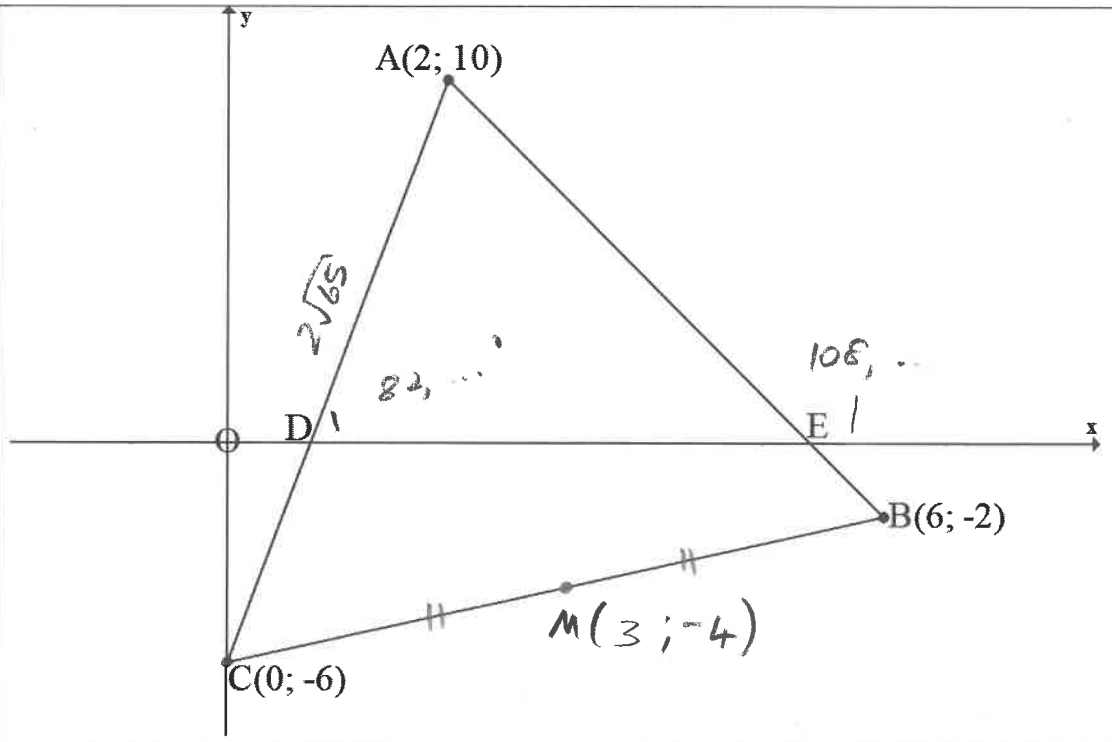
$-40 = -6(x+3)$ ✓ $(x \neq -3)$

$\frac{11}{3} = x$ ✓

5

(5)

**3.2**




**3.2.1**

$x_M = \frac{0+6}{2} \qquad y_M = \frac{-6+(-2)}{2}$

$= 3 \qquad = -4$

$\therefore M(3; -4)$ ✓ ✓

2

(2)

A(2;10) B(6;-2) C(0;-6)



3.2.2

$AC = \sqrt{(-6-10)^2 + (0-2)^2}$ ✓

$= \sqrt{260}$

$= \sqrt{4 \times 65}$

$= 2\sqrt{65}$ ✓

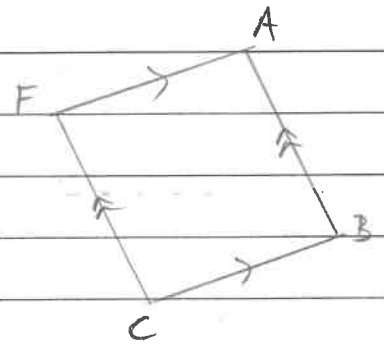
2

(2)

3.2.3

$B(6;-2) \xrightarrow{\substack{4 \leftarrow \\ 12 \uparrow}} A(2;10)$

$C(0;-6) \xrightarrow{\substack{4 \leftarrow \\ 12 \uparrow}} F(-4;6)$ ✓✓




2

(2)

3.2.4

$m_{AB} = \frac{10-(-2)}{2-6} = -3$ ✓

$m_{BC} = \frac{-2-(-6)}{6-0} = \frac{2}{3}$ ✓

$m_{BC} \times m_{AB} = \frac{2}{3} \times -3 = -2$

$\neq -1$ ✓

3

$\therefore BC \perp AB$

$\therefore ABCF$ NOT rectangle no int $\angle = 90°$

(3)

**3.2.5**

$m_{AB} = -3$ ✓

$y = -3x + c$

Sub $A(2;10)$

$10 = -3(2) + c$ ✓

$16 = c$

$\therefore y = -3x + 16$ ✓ AB

$m_{AC} = \frac{10-(-6)}{2-0} = 8$

$y = 8x + c$

Clearly $c = -6$ ✓ ✓

$\therefore y = 8x - 6$ AC

5

(5)

**3.2.6**

$\tan \hat{D}_1 = 8$ ✓

ref $\hat{} = 82,87...^\circ$

tan + in

I: $\hat{D}_1 = 82,87...^\circ$ ✓

$\tan \hat{E}_1 = -3$ ✓

ref $\hat{} = 71,56...^\circ$

tan − in

II: $\hat{E}_1 = 108,43...^\circ$ ✓

$\hat{A} + 82,87...^\circ = 108,43...^\circ$ ext $\hat{} \Delta$

$\hat{A} = 25,56^\circ$ ✓ ✓

6

(6)

**3.2.7**

DE:

$x_{int}$ $0 = 8x - 6$ $x_{int}$ $0 = -3x + 16$

$\frac{3}{4} = x_D$ ✓ $\frac{16}{3} = x_E$ ✓ 5,33...

$\therefore DE = \frac{16}{3} - \frac{3}{4} = \frac{55}{12}$ ✓ 4,583...

0,75

$h: y_A = 10$ ✓

area $= \frac{1}{2}\left(\frac{55}{12}\right) \times 10 = \frac{275}{12}$ ✓ $u^2$ 22,92

5

(5)

**[30]**

## QUESTION 4/*VRAAG 4*

**4.1**

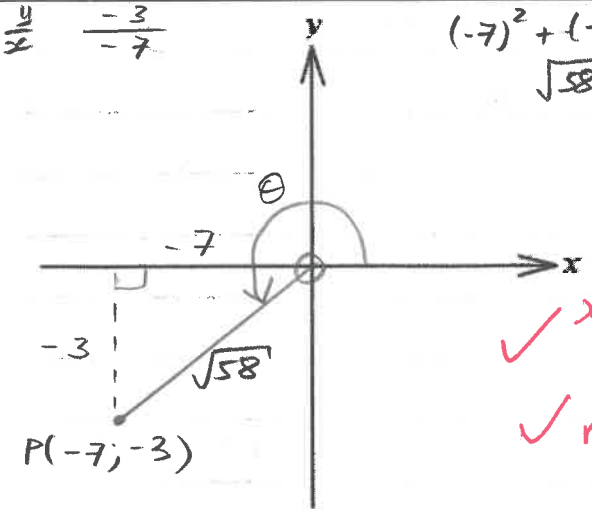
$\tan\theta = \frac{3}{7}$ $\frac{y}{x}$ $\frac{-3}{-7}$

I III

$\cos < 0$

II III

Quad ✓




$(-7)^2 + (-3)^2 = r^2$ Pyth

$\sqrt{58} = r$

✓ x y

✓ r

$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{58}}$

$\cos\theta = \frac{x}{r} = \frac{-7}{\sqrt{58}}$

$\therefore \frac{\sin\theta + \cos\theta}{2\sin\theta} = \frac{\frac{-3}{\sqrt{58}} + \left(\frac{-7}{\sqrt{58}}\right)}{2 \times \left(\frac{-3}{\sqrt{58}}\right)} = \frac{\frac{-3}{\sqrt{58}} - \frac{7}{\sqrt{58}}}{\frac{-6}{\sqrt{58}}}$

$= \frac{-10}{\sqrt{58}} \times \frac{\sqrt{58}}{-6}$

$= \frac{5}{3}$ ✓

6

(6)

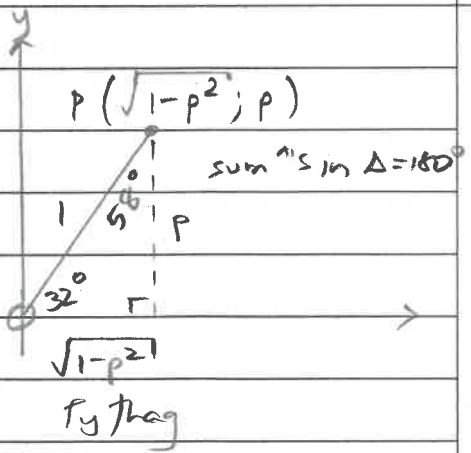
**4.2.1**

$\sin 32^\circ = p$ $\frac{p}{1}$ $\frac{y}{r}$

$\tan(-32^\circ)$

$= -\tan 32^\circ$ ✓

$= -\frac{p}{\sqrt{1-p^2}}$ ✓ ✓ $\frac{y}{x}$




3

(3)

**4.2.2**

$\sin 418^\circ = \sin 58^\circ$ ✓ $\quad -360^\circ$

$= \frac{\sqrt{1-p^2}}{1}$ $\quad \frac{o}{h}$

$= \sqrt{1-p^2}$ ✓ CA

2

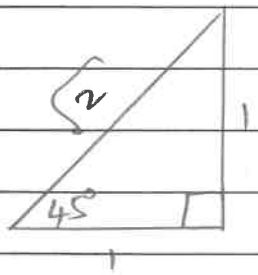
(2)

**4.3**

$\frac{\sin 120^\circ \tan 300^\circ}{\cos(-60^\circ)\tan 225^\circ}$

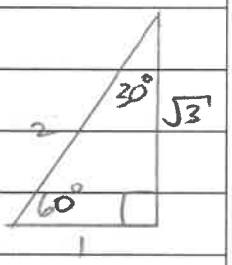
• $\sin 120^\circ$
$= \sin(180^\circ - 60^\circ)$
$= +\sin 60^\circ$ ✓
$= \frac{\sqrt{3}}{2}$

• $\tan 300^\circ$
$= \tan(360^\circ - 60^\circ)$
$= -\tan 60^\circ$ ✓
$= -\frac{\sqrt{3}}{1}$




• $\cos(-60^\circ)$
$= +\cos 60^\circ$ ✓
$= \frac{1}{2}$

• $\tan 225^\circ$
$= \tan(180^\circ + 45^\circ)$
$= +\tan 45^\circ$ ✓
$= \frac{1}{1}$




+ diagram

$\therefore -\frac{\left(\frac{\sqrt{3}}{2}\right)(-\sqrt{3})}{\left(\frac{1}{2}\right)(1)}$ ✓ $= \frac{-\frac{3}{2}}{\frac{1}{2}}$

+ diagram

$= -\frac{3}{2} \times \frac{2}{1}$

$= -3$ ✓

7

(7)

**4.4**

$$\frac{\cos^2 x - \cos x - \sin^2 x}{2\sin x \cos x + \sin x}$$

$$= \frac{\cos^2 x - \cos x - (1 - \cos^2 x)}{2\sin x \cos x + \sin x}$$

$$= \frac{\cos^2 x - \cos x - 1 + \cos^2 x}{2\sin x \cos x + \sin x}$$

$$= \frac{2\cos^2 x - \cos x - 1}{2\sin x \cos x + \sin x}$$

$$= \frac{(\cos x - 1)(2\cos x + 1)}{\sin x(2\cos x + 1)}$$

$$= \frac{\cos x - 1}{\sin x}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\frac{1}{\tan x} - \frac{1}{\sin x}$$

$$= \frac{1}{\frac{\sin x}{\cos x}} - \frac{1}{\sin x}$$

$$= 1 \times \frac{\cos x}{\sin x} - \frac{1}{\sin x}$$

$$= \frac{\cos x}{\sin x} - \frac{1}{\sin x}$$

$$= \frac{\cos x - 1}{\sin x}$$

5

(5)

**4.5**

$\sin 5\theta = \cos(\theta - 40^\circ)$ $\quad A = 5\theta \quad B = \theta - 40^\circ$

$\sin A = \cos B$

$\sin(90^\circ - B)$ I $\quad \sin(90^\circ + B)$ II

$\sin A = \sin(90 - B)$ or $\sin A = \sin(90^\circ + B)$

$A = 90^\circ - B + k \cdot 360^\circ$

$5\theta = 90^\circ - (\theta - 40^\circ) + k \cdot 360^\circ$

$5\theta = 90^\circ - \theta + 40^\circ + k \cdot 360^\circ$

$6\theta = 130^\circ + k \cdot 360^\circ$

$\theta = 21{,}67^\circ + k\,60^\circ; k \in Z$

$\frac{65}{3}^\circ$

$A = 90^\circ + B + k \cdot 360^\circ$

$5\theta = 90^\circ + (\theta - 40^\circ) + k \cdot 360^\circ$

$5\theta = 90^\circ + \theta - 40^\circ + k \cdot 360^\circ$

$4\theta = 50^\circ + k \cdot 360^\circ$

$\theta = 12{,}5^\circ + k\,90^\circ; k \in Z$

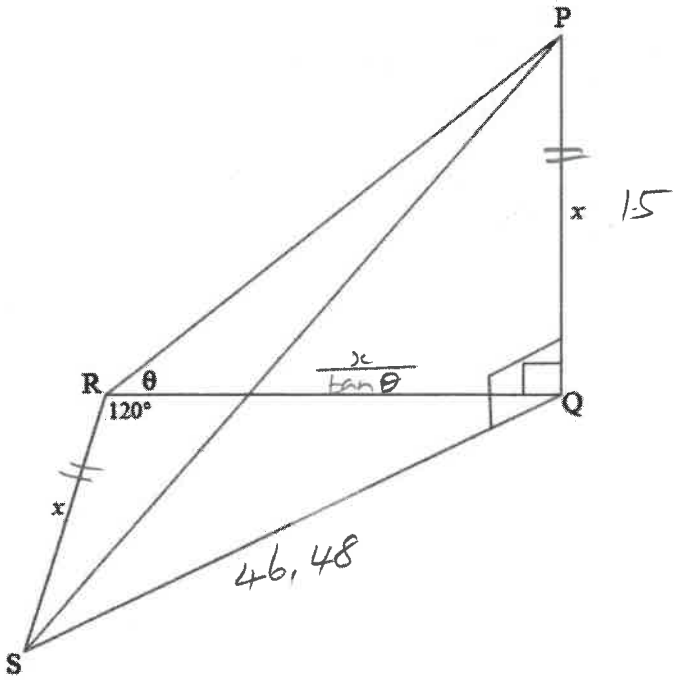
$\frac{25}{2}^\circ$

6

(6)

**[29]**

**QUESTION 5/*VRAAG 5***




**5.1**

$\Delta PRQ$

$\frac{x}{QR} = \tan\theta$ ✓

$x = QR\tan\theta$

$\frac{x}{\tan\theta} = QR$ ✓

2

(2)

**5.2**

$\Delta QRS$

$QS^2 = x^2 + \left(\frac{x}{\tan\theta}\right)^2 - 2x\left(\frac{x}{\tan\theta}\right)\cos 120°$

$= x^2 + \frac{x^2}{\tan^2\theta} - \frac{2x^2}{\tan\theta}\cdot\left(-\frac{1}{2}\right)$

$= x^2 + \frac{x^2}{\tan^2\theta} + \frac{x^2}{\tan\theta}$ ✓

$= x^2\left(1 + \frac{1}{\tan^2\theta} + \frac{1}{\tan\theta}\right)$ ✓

$QS = \sqrt{x^2\left(\frac{1}{\tan^2\theta} + \frac{1}{\tan\theta} + 1\right)}$

$= \sqrt{x^2}\sqrt{\frac{1}{\tan^2\theta} + \frac{1}{\tan\theta} + 1}$

$= x\sqrt{\frac{1}{\tan^2\theta} + \frac{1}{\tan\theta} + 1}$

4

(4)

5.3.1

$$QS = 15\sqrt{\frac{1}{\tan^2 22^\circ} + \frac{1}{\tan 22^\circ} + 1}$$ ✓

$$= 46,48$$ ✓

2

(2)

5.3.2

$\Delta QPS$: $\tan Q\hat{P}S = \frac{46,48}{15}$ ✓ $\frac{o}{a}$

ref $\hat{}$ = 72,11 ...

tan + in $\hat{}$

I: $Q\hat{P}S = 72,11^\circ$ ✓

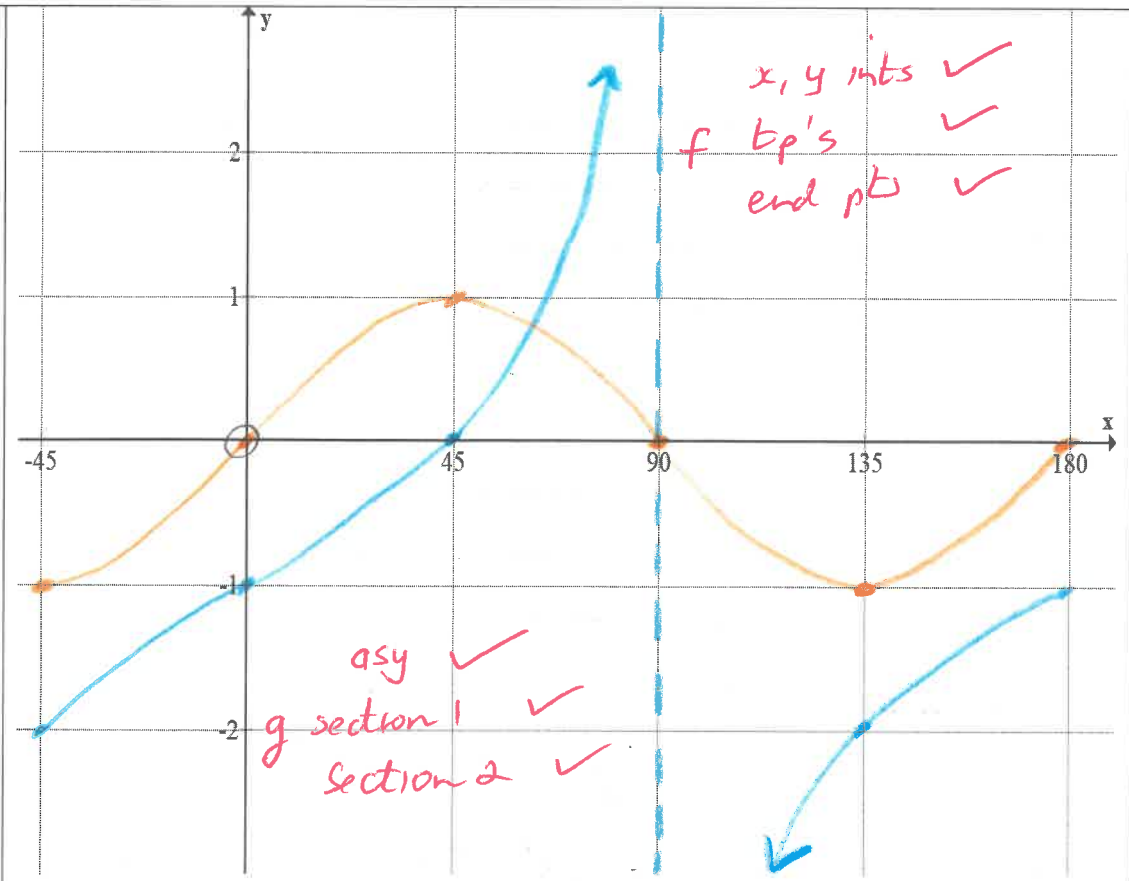
2

(2)

**[10]**

**QUESTION 6/*VRAAG 6***

6.1




$y = \sin^2 x$ f

g $y = \tan x - 1$ $y = \tan(x) - 1$ NB

(6)

6.2.1 $\sin 2x > 0$ $x \in (0°; 90°)$ ✓

$y_f$ +

1

(1)

6.2.2 $f(x)\ g(x) \geq 0$

$y_f \cdot y_g$ $\circ$ +

$x \in [-45°; 0°]$ or $[45°; 90°)$ or $(90°; 180°]$ ✓ ✓ ✓

3

(3)

6.3 $R_f$ : $y \in [-1; 1]$

+2 : $y \in [1; 3]$ ✓ ✓

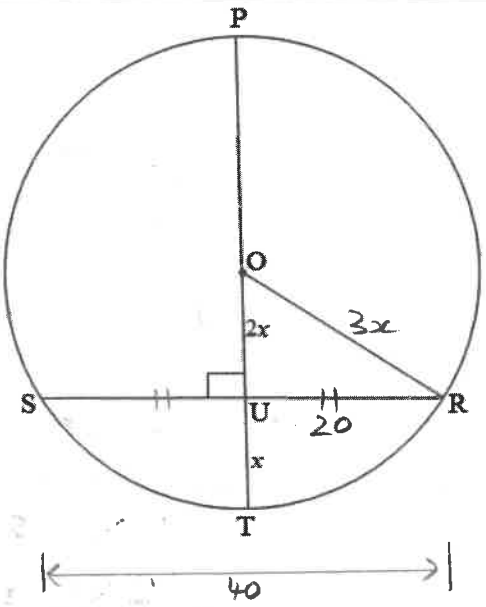
2

(2)

**[12]**

## QUESTION 7/*VRAAG 7*

7.1




7.1.1

OR = 3x ✓S radii

1

(1)

7.1.2

UR = 20 ✓S ✓R line from centre ⊙ ⊥ to chord

$(2x)^2 + (20)^2 = (3x)^2$ ✓SR Pythag

$4x^2 + 400 = 9x^2$ ✓S

$400 = 5x^2$

$80 = x^2$

$\sqrt{80} = x$

$\sqrt{16.5} =$

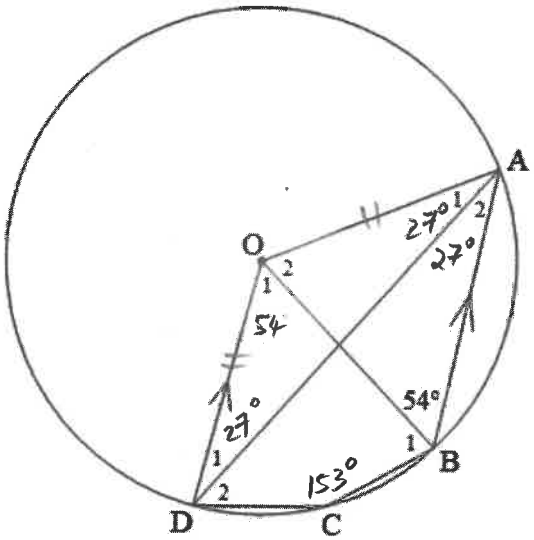
$4\sqrt{5} = x$ ✓S

$(4\sqrt{5}$ cm)

5

(5)

**7.2**




**7.2.1**

$\hat{O}_1 = 54°$ ✓S ✓R alt ^'s =, OD ∥ AB

$\hat{A}_2 = 27°$ ✓S ✓R ^@ centre = 2× ^@ circumf

4 (4)

**7.2.2**

$\hat{C} = 153°$ ✓S ✓R opp ^'s cyclic quad = 180°

2 (2)

**7.2.3**

$\hat{D}_1 = 27°$ ✓ alt ^'s =, OD ∥ AB

DO = OA radii

$\therefore \hat{A}_1 = 27°$ ✓✓ ^'s opp = sides

$\therefore \hat{A}_1 = \hat{A}_2$ both = 27°

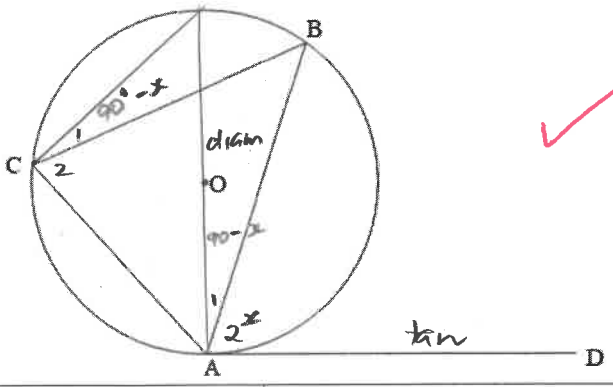
$\therefore$ DA bisects $\hat{OAB}$

3 (3)

**[15]**

## QUESTION 8/*VRAAG 8*

**8.1**



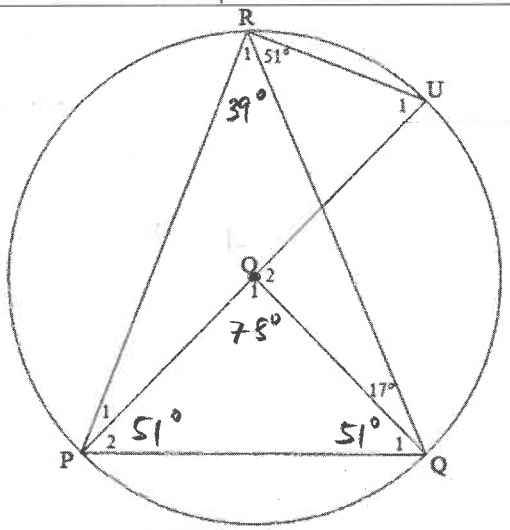

✓ constr

| | | |
|---|---|---|
| let $\hat{A}_2 = x$ | | |
| $\therefore \hat{A}_1 = 90° - x$ ✓S ✓R | tan ⊥ rad | |
| $\therefore \hat{C}_1 = 90° - x$ ✓SR | ^'s in same ⊙ segm = | |
| $\therefore \hat{C}_2 = x$ ✓SR | ^ in semi ⊙ = 90° | |
| $\therefore \hat{A}_2 = \hat{C}_2$ | both = x | |
| $\therefore D\hat{A}B = B\hat{C}A$ | | |
| | | |
| | | |
| | | |

5

(5)

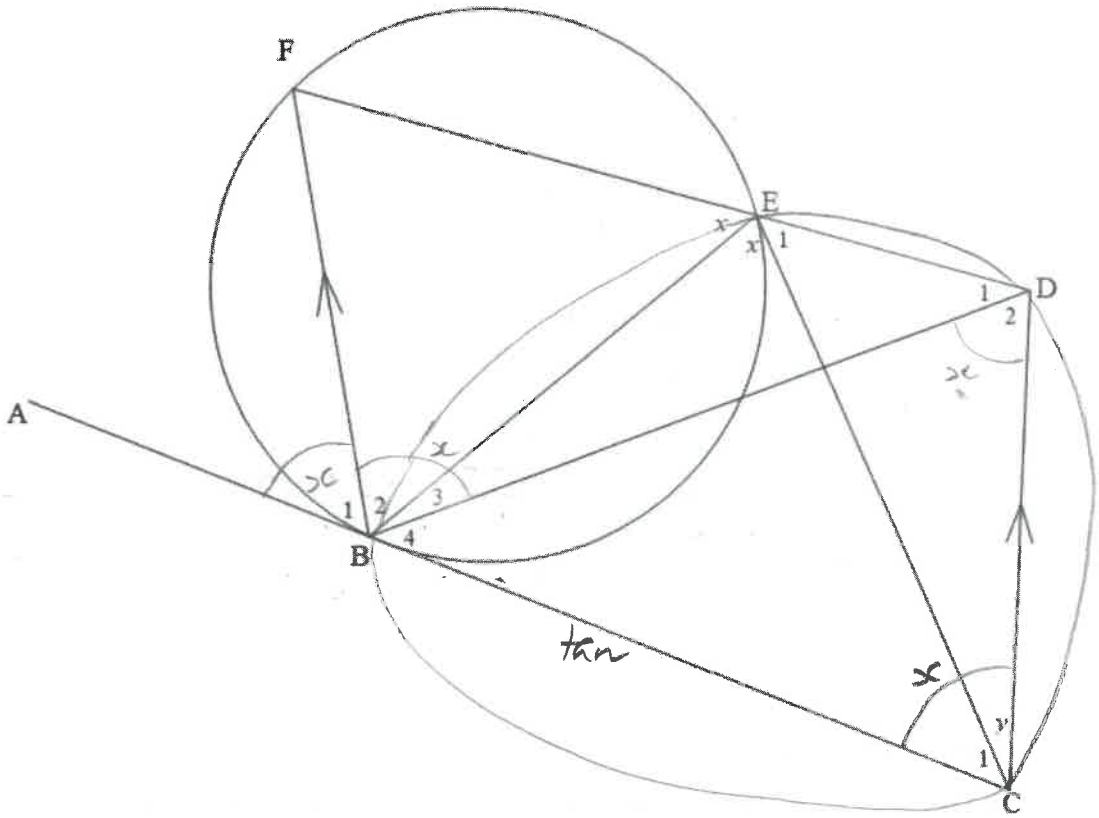
**8.2**




| | | | |
|---|---|---|---|
| 8.2.1 | $\hat{R}_1 = 39°$ ✓S ✓R | ^ in semi ⊙ = 90° | (2) |
| 8.2.2 | $\hat{O}_1 = 78°$ ✓S ✓R | ^@ centre = 2x ^@ circumf | (2) |
| 8.2.3 | $\hat{Q}_1 = 51°$ ✓S ✓R | radii ^'s opp = sides<br>sum ^'s in Δ = 180° | (2) |
| 8.2.4 | $\hat{U}_1 = 68°$ ✓S ✓R | ^'s in same ⊙ segm = | (2) |
| | | | [13] |

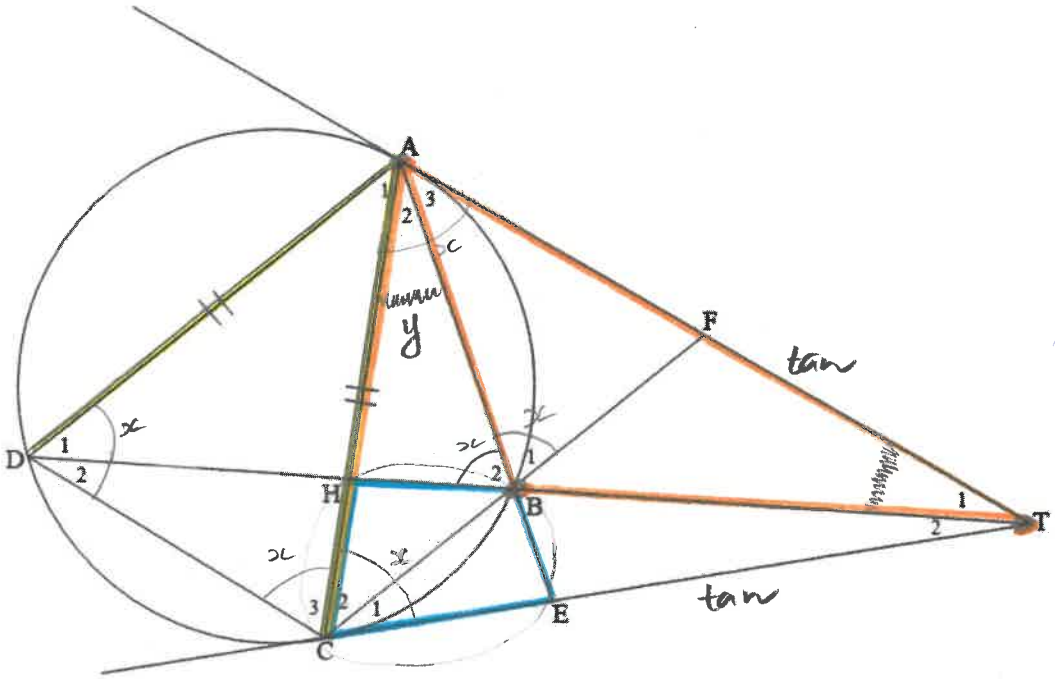
8

**QUESTION 9/*VRAAG 9***




| | | | |
|---|---|---|---|
| 9.1.1 | $A\hat{B}F = x$ ✓S ✓R | tan chord thm | 2 (2) |
| 9.1.2 | $B\hat{C}D = x$ ✓S ✓R | Corr ∠s =, FB ∥ DC | 2 (2) |
| 9.2 | $F\hat{E}B = B\hat{C}D$ ✓R | Conv ext ∠ cyclic quad | 1 (1) |
| 9.3 | $\hat{D}_2 = x$ ✓S ✓R | ∠s in same ⊙ segm = | 4 (4) |
| | $\hat{B}_2 + \hat{B}_3 = x$ ✓S ✓R | alt ∠s =, FB ∥ DC | |
| | | | |
| | | | |
| | | | |
| | | | [9] |

**QUESTION 10/*VRAAG 10***




| | | | |
|---|---|---|---|
| 10.1 | let $\hat{B}_1 = x$ | | |
| | $\therefore \hat{D}_1 + \hat{D}_2 = x$ ✓S ✓R | ext ∠ cyclic quad | |
| | $\therefore \hat{C}_3 = x$ ✓SR | ∠'s opp = sides | 4 |
| | $\therefore \hat{B}_2 = x$ ✓SR | ∠'s in same ⊙ segm = | |
| | $\therefore \hat{B}_1 = \hat{B}_2$ | both = x | |
| | | | (4) |
| 10.2 | $\hat{C}_1 + \hat{C}_2 = x$ ✓SR | tan chord thm | |
| | $\therefore \hat{B}_2 = \hat{C}_1 + \hat{C}_2$ ✓S | both = ∠c | 3 |
| | $\therefore$ BECH is a cyclic quad ✓R | conv ext ∠ cyclic quad | |
| | | | (3) |

| | | | |
|---|---|---|---|
| 10.3 | Let $\hat{A}_2 = y$ | | |
| | $TA = TC$ ✓SR | tan's from ext common pt = | |
| | $\therefore y + \hat{A}_3 = x$ ✓SR | ^is opp = sides | |
| | $y = x - \hat{A}_3$ | | |
| | $\hat{A}_3 + T_1 = x$ ✓SR | ext ^ Δ (ABT) | 5 |
| | $\therefore T_1 = x - \hat{A}_3$ | | |
| | $\therefore \hat{T}_1 = y$ | both = $x - A_3$ | |
| | $\therefore \hat{A}_2 = \hat{T}_1$ ✓S | both = y | |
| | $\therefore$ CA is a tan ✓R | conv tan chord thm | |
| | ⊙ ABT ⟹ | | (5) |
| | | | [12] |

**TOTAL/*TOTAAL*: 150**